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P.R. Krishnaiah and C.R. Rao

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A summary of the work done under the contract is reported here. The work involved a broad spectrum of topics in the area of multivariate analysis. These topics include contingency tables, distribution theory, selection of variables, classification and pattern recognition and statistical inference. cont; ; words include

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MULTIVARIATE ANALYSIS AND ITS APPLICATIONS:
Final Report on the Contract F49620-82-K-0001
Sponsored by the Air Force Office
of Scientific Research

P. R. Krishnaiah and C. R. Rao
Center for Multivariate Analysis

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This is the final report on the work done during the period of December 15, 1981-September 30, 1984 under the Contract F49620-82-K-0001 sponsored by the Air Force Office of Scientific Research. Various reports and papers written under this contract are divided into three parts. The work in these papers is supported partially or completely by the above Contract.

In Part I, we give a list of papers which were issued as technical reports of the Center for Multivariate Analysis (CMA). All the technical reports were submitted for publication in journals and other formal publications. Many of them were accepted for publication and the remaining papers are still under consideration for possible publication.

In Part II, we have listed the papers which were published or accepted for publication in journals and books. Most of the papers in Part II were already issued as CMA technical reports earlier.

In Part III, we have included a list of papers which were not included in Parts I and II.

The work done under this contract covers a wide spectrum of topics in the area of multivariate analysis. The topics covered include, among other things, classification and related topics, contingency tables, inequalities, large dimensional random matrices, reliability of complicated systems, robustness of test procedures, selection of important variables, etc. This work has applications in many areas of interest to the Air Force.

Some highlights of the work are given below.

1. CLASSIFICATION AND RELATED TOPICS

A new measure of entropy called quadratic entropy has been intorduced and its applications have been studied. it provides a unified measure of diversity (variability) for both qualitative and quantitative measurements. This has

opened up several possibilities, of measuring diversity with respect to a complex of measurements, of apportioning diversity in a hierarchical classification, estimating diversity caused by different factors or causes and testing their significance. The analysis can be applied to practical problems to identify and control sources of variation with respect to multiple measurements.

Based on quadratic entropy, distance measures between populations have been developed for use in cluster analysis. New methods of cluster analysis are proposed using complete subgraphs and applied to some practical problems.

Some work is also done in the area of classification when some variables are discrete and other variables are continuous.

The techniques of classification have many important applications. Discussions some of these applications are given in the following book:

> P. R. Krishnaiah and L. N. Kanal (Editors). Classification, Pattern Recognition and Reduction of Dimensionality. North-Holland Publishing Company, 1982.

For example, these techniques are useful in identifying a signal in presence of noise on the basis of the data on various variables (or parameters) of the signal. They are useful in identifying targets and patterns. The classification techniques can be also used in the area of speech recognition. Speech recognition is very useful in identifying the voice of the pilots and in research in robotics. The techniques of classification are also useful in medical diagnosis of pilots. In recruitment of pilots, Air Force is interested in finding out whether a particular candidate has the potential to be a good pilot. The techniques developed for classification and prediction will be useful for this purpose.

2. SELECTION OF IMPORTANT VARIABLES

Many times, one is confronted with the problem of prediction and forecasting (like meteorological forecasts) on the basis of the data on a large number of variables In these situations, it is of interest to select a small number of important variables from cost consideration. Also, sometimes unimportant variables may distort information. Some work was done in the area of selection of variables.

New techniques have been proposed for simultaneous prediction (predicting in a number of independent cases) which have greater efficiency in the long run than those based on individual predictions.

3. ROBUSTNESS OF TEST PROCEDURES

Many test procedures in the area of multivariate analysis are based upon the assumption that the distributions underlying the data are multivariate normal. Asymptotic distributions of a wide class of test statistics on covariance matrices and correlation matrices are derived when the underlying distribution is a mixture of multivariate normal distributions. When the null hypothesis is true, the robustness of a wide class of test statistics is established under the multivariate analysis of variance model and canonical correlation model.

4. RELIABILITY OF COMPLICATED SYSTEMS

Weibull, Pareto, exponential and stable distributions have extensive applications in reliability theory, life testing and queing. Several characterizations of these distributions are available, one of which is generally described as lack of memory. These distributions are now characterized by a weaker property called the integrated lack of memory which may be stated as follows:

> "The conditional distribution of the excess life of a component of a system given that it survives some component of an independent system is the same as the unconditional distribution of the component."

Such a characterization is useful in determining the nature of life distributions based on conditional survival data, and has several practical applications in engineering reliability of complicated multicomponent systems.

It is not always relistic to assume that the performances of various components are independent of each other. This assumption is relaxed and various multivariate gamma and multivariate gamma-Weibull distributions are considered to describe the joint distributions of the failure times of components. The assumption of independence between shocks (or time points between minimal repairs) is not always realistic. This assumption is also relaxed and some multivariate distributions are considered to describe the joint performance of various components. This work is useful in investigating the reliability of not only binary systems but also multistate systems.

5. LARGE DIMENSIONAL RANDOM MATRICES

There are several practical situations when the experimenter is confronted with the analysis of multivariate data when the number of variables is very large. The exact distributions of some test statistics based upon the eigenvalues of the random matrices are known and they are computed for small number of variables. But, there are many test statistics whose exact distributions are either unknown or complicated to compute when the number of variables are large. Considerable amount of work was done in the literature in deriving the asymptotic distributions of test statistics when the sample size is very large. Very little work is done when the number of

variables is also large. Under this Contract, asymptotic distributions are derived when (i) sample size is very large and (ii) sample size and number of variables are large.

6. REDUCTION OF DIMENSIONALITY

If there are several multivariate populations it is important to examine whether the mean values exhibit a simple structure. Such a study will be of great help in interpreting the mean values and also in constructing more efficient classification procedures. Using a FANOVA (factor analysis of variance model) type model, tests have been constructed for a given complexity of the interaction terms for the mean values when the variance covariance matrices are completely general and unknown, and when they are of the reducible type. Further theoretical work is being pursued and practical applications are being considered.

Part I: LIST OF TECHNICAL REPORTS WITH ABSTRACTS

1. Krishnaiah, P. R. Selection of Variables Under Univariate Regression Models. Technical Report #82-01, Center for Multivariate Analysis, April, 1982.

   In this paper, the author reviews some methods of selection of variables under univariate regression models. The methods reviewed are forward selection, stepwise regression, backward elimination, overall F test and finite intersection tests (FIT). Some of the drawbacks of the first three methods are also discussed. Also, the author discusses some of the merits of the FIT over the overall F test.

2. Sinha, B. K. and Sarkar, S. K. Invariant Confidence Sequences for Some Parameters in a General Linear Regression Model. Technical Report #82-02. Center for Multivariate Analysis, April 1982.

   Let $X_1, X_2, \ldots$ be independent p-variate normal vectors with $E X_\alpha = \beta Y_\alpha$, $\alpha = 1,2,\ldots$ and same p.d. dispersion matrix Σ. Here $\beta : p \times q$ and Σ are unknown parameters and Y_α's are known $q \times 1$ vectors. Writing $\beta = (\beta_1' \ \beta_2')' = (\beta_{(1)} \ \beta_{(2)})$ with $\beta_i : p_i \times q(p_1 + p_2 = p)$ and $\beta_{(i)}: p \times q_i(q_1 + q_2 = q)$, we have constructed invariant confidence sequences for (i) β, (ii) $\beta_{(1)}$, (iii) $\beta_{(1)}$ when $\beta_2 = 0$ (iv) $\sigma^2 = |\Sigma|$. This uses the basic ideas of Robbins (1970) and generalizes some of his and Lai's (1976) results. In the process alternative simpler solutions of some of Khan's results (1978) are obtained.

3. Rao, C. R. Measures of Diversity and Applications. Technical Report #82-06. Center for Multivariate Analysis, April 1982.

   Two general methods of obtaining measures of diversity within a population are discussed. One is based on an intrinsic notion of dissimilarity between individuals and the other makes use of the concepts of entropy.

   Methods for apportionment of diversity in a hierarchically classified set of populations are discussed. An example is given using genetic data.

   The concept of analysis of diversity as a generalization of analysis of variance is developed for populations classified by combinations of different levels of chosen factors. The method is illustrated with an analysis of some data.

4. Rao, C. R. Deny's Theorem and Its Application to Characterizations of Probability Distributions. Technical Report #82-07. Center for Multivariate Analysis, April 1982.

   A theorem of Deny is stated and its applications to a number of characterization problems of probability distributions are given. During the last fifteen years, a number of characterizations of the exponential, Weibull, stable, Pareto, and geometric distributions have been considered by a number of authors. The proofs given in the different cases range from applications of

function theory and Laplace transform to real variable techniques and differential equations. A special case of Deny's theorem recently proved by Lau and Rao (1982 a,b), and a certain discrete version of the Lau-Rao formulation proved earlier by Shanbhag (1977) provide a unified approach to all these problems and leads in many cases to stronger results.

5. Schmidhammer, J. L. On the Selection of Variables Under Regression Models Using Krishnaiah's Finite Intersection Tests. Technical Report #82-08. Center for Multivariate Analysis, April 1982.

The application of Krishnaiah's Finite Intersection Test to the selection of variables in univariate and multivariate statistical regression is examined. The multivariate F distribution is defined, and several probability inequalities useful for the calculation of critical values in the multivariate F distribution are reviewed. The use of the procedure is illustrated on both a univariate and a multivariate set of data.

6. Fang, C. and Krishnaiah, P. R. Asymptotic Distributions of Functions of the Eigenvalues of the Doubly Noncentral Multivariate F Matrix. Technical Report #82-10. Center for Multivariate Analysis, May 1982.

In this paper, the authors derive asymptotic expressions for joint distributions of certain functions of the eigenvalues of the doubly noncentral multivariate F matrix. These expressions are in terms of the multivariate normal density and multivariate Hermite polynomials. Some applications of the above results are also discussed.

7. Sampson, A. R. and Block, H. W. Inequalities on Distributions - Bivariate and Multivariate. Technical Report #82-11. Center for Multivariate Analysis, May 1982.

Inequalities concerning bivariate and multivariate distributions in statistics are surveyed, as well as historical background. Subject treated include inequalities arising through positive and negative dependence; Boole, Bonferroni, and Fréchét inequalities; convex symmetric set inequalities; stochastic ordering; stochastic majorization and inequalities obtained by majorization; Chebyshev and Kolmogorov-type inequalities; multivariate moment inequalities; and applications to simultaneous inference, unbiased testing and reliability theory.

8. Ghosh, J. K. and Sinha, B. K. Third Order Efficiency of the MLE - A Counterexample. Technical Report #82-13. Center for Multivariate Analysis, June 1982

We give an example of a curved exponential where the maximum likelihood estimate is not third order efficient either in the sense of Fisher-Rao or Rao.

9. Krishnaiah, P. R. Selection of Variables in Discriminant Analysis. Technical Report #82-15. Center for Multivariate Analysis, June 1982.

In this paper, the author gives a review of some techniques for the selection of variables in discriminant analysis.

10. Nguyen, T. T. and Sampson, A. R. Extreme Points of Bivariate Discrete Distributions with Fixed Marginals. Technical Report #82-19. Center for Multivariate Analysis, July 1982.

The extreme points of fixed-marginal bivariate discrete probability distributions are characterized, thereby generalizing Birkhoff's Theorem. For the finite case, a constructive algorithm which generates all extreme points is given. The relationships between the Fréchet bounds and other extreme points are discussed. Within the set of extreme points, the upper and lower Frechet bounds are characterized by their respective TP_2 and RR_2 properties.

11. Nguyen, T. T. and Sampson, A. R. Counting the Number of $p \times q$ Integer Matrices More Concordant Than a Given Matrix. Technical Report #82-20. Center for Multivariate Analysis, July 1982.

A method for generating and counting the set of matrices of nonnegative integer elements with fixed row and column sums based on the "more concordant" partial ordering is given. A counting formula is presented in the general case. A simple expression is given for $C(r_1,r_2,r_3;r_1,r_2,r_3)$, the cardinality of the set of 3×3 matrices with row sum (r_1,r_2,r_3) and column sum (r_1,r_2,r_3).

12. Nguyen, T. T. and Sampson, A. R. The Geometry of Certain Fixed Marginal Probability Distributions. Technical Report #82-21. Center for Multivariate Analysis, July 1982.

The geometry of the set of $p \times q$ p.m.f. matrices with fixed marginals is discussed. The positively quadrant dependent and negatively quadrant dependent subsets are also considered. Explicit graphical representations of these sets are given in the 2×2 and 2×3 cases.

13. Rao, C. R. Multivariate Analysis: Some Reminiscences on its Origin and Development. Technical Report #82-23. Center for Multivariate Analysis, August 1982.

The article traces the history of multivariate analysis, the pioneering contributions of R. A. Fisher, the development of multivariate statistical methodology for applications in various fields under the guidance of Fisher, the work of the Indian School of Statisticians under the leadership of P. C. Mahalanobis, the author's experience in the statistical analyses of three large bodies of anthropometric data, and some of the modern trends of research. The origin and development of multivariate analysis clearly show that contact with live problems is essential for worthwhile research in statistical methodology.

14. Kimeldorf, G. and Sampson, A. R. Positive Dependence Orderings. Technical Report #82-24. Center for Multivariate Analysis, September 1982.

    This paper presents a systematic basis for studying orderings of bivariate distributions according to their degree of positive dependence. The general concept of a positive dependence ordering (PDO) is introduced and its properties discussed. A number of existing orderings are evaluated as to whether or not each is a PDO. A new ordering according to the degree of total positivity of order two (TP_2) is given, and is shown to be a PDO. This TP_2 PDO is a subordering of the more positively quadrant dependent PDO.

15. Rao, C. R. Matrix Derivatives: Applications in Statistics. Technical Report #82-25. Center for Multivariate Analysis, September 1982.

    Rules are given for computing simultaneously all the partial derivatives of scalar function f(X) of a matrix argument X with respect to the elements of X. The partial derivatives are expressed in a matrix form. Methods are given for deriving matrix derivatives for functions of the form g[f(X)], f(X)g(X), $f(X^{-1})$ and so on.

16. Rao, C. R. and Kleffe, J. Mixed Linear Models. Technical Report #82-26. Center for Multivariate Analysis, September 1982.

    Starting with the general linear model $Y = X\beta + \varepsilon$ where $E(\varepsilon\varepsilon') = \theta_1 V_1 + \ldots + \theta_p V_p$, the theory of minimum norm quadratic estimation (MINQE) of the parameter $\theta = (\theta_1, \ldots, \theta_p)'$ is developed. The method depends on the choice of a natural quadratic estimator of θ in terms of the unobservable variable ε and comparing it with a quadratic estimator Y'AY in terms of the observable variable Y. The matrix A is determined by minimizing the difference between two quadratic forms. By placing restrictions on Y'AY such as unbiasedness (U), invariance under translation of Y by $X\beta$, (I), non-negative definiteness (D), different kinds of MINQE's such as MINQE(I), MINQE(U), MINQE(U,D,), etc. are generated.

    A class of iterated MINQE's (MINQE's) is developed to obtain estimators free from a priori information used for the construction of MINQE's. This class is shown to include maximum likelihood (ML) and marginal ML (MML) estimators. Thus, the MINQE principle provides a unified theory of estimation of variance components.

17. Sinha, B. K. and Drygas, H. Robustness Properties of the F-test and Best Linear Unbiased Estimators in Linear Models. Technical Report #82-28. Center for Multivariate Analysis, November 1982.

    Consider a linear model $Y = X\beta + \sigma\varepsilon$, $E(\varepsilon) = 0$, $E(\varepsilon\varepsilon') = I_n$ with β, σ unknown. For the problem of testing the linear hypothesis $C\beta = \delta$, im(C') im(X'), Ghosh and Sinha (1980) proved that the properties of the usual F-test being LRT and UMPI (under a suitable group of transformations) remain valid for specific

non-normal families. In this paper it is shown that both criterion and inference robustness of the F-test hold under the assumption ε $q(\varepsilon'\varepsilon)$, q convex and isotonic. This result is similar to robustness property of Hotelling's T^2-test proved by Kariya (1981). Finally, it is proved that the Best Linear Unbiased Estimator (BLUE) of any estimable function $C\beta$ is more concentrated around $C\beta$ than any other unbiased estimator of $C\beta$ under the assumption that ε is spherically distributed.

18. Drygas, H. On the Kleffe-Approach for Computing Covariance-Operator of Tensor-Products. Technical Report #82-29. Center for Multivariate Analysis, October 1982.

Let ε be a vector of expectation zero, with independently, identically quasi-normally distributed components. It is the purpose of this paper to compute the covariance-operators $\varepsilon \otimes 3$ and $\varepsilon \otimes 4$ via a technique developed by J. Kleffe. It turns out that the computations are still tedious, but a decomposition of the combinatorial problems involved is taking place.

19. Drygas, H. Linear Sufficiency and Some Applications in Multilinear Estimation. Technical Report #82-30. Center for Multivariate Analysis, October 1982.

In the linear model $Y = X\beta + u$ the question arises when a linear transformation $Z = LY$ contains all information of the linear model. This problem was solved by Baksalary and Kala (Annals 1981), Drygas (Sankhyā, forthcoming) and J. Muller (Ph.D. thesis, Kassel 1982). As an application we consider the estimation of the variance of the observations, its skewness and its kurtosis. This is done by considering so-called derived models. (Anscombe, Pukelsheim, Kleffe). Linear sufficient statistics are derived for these problems.

20. Müller, J. Sufficiency and Completeness in the Linear Model. Technical Report #82-32. Center for Multivariate Analysis, October 1982.

This paper provides further contributions to the theory of linear sufficiency and linear completeness. The notion of linear sufficiency was introduced in Baksalary, J. K. and R. Kala (1981) (Linear Transformations preserving the best linear unbiased estimator in a general Gauss-Markov model. Ann. Stat. 9, 913-916) and Drygas, H. (Sufficiency and completeness in the general Gauss-Markov model. Submitted to Sankhyā) with respect to the linear model $Ey = X\beta$, var $y = V$. In addition to correcting an inadequate proof of Drygas, the relationship to an earlier definition and to the theory of linear prediction is also demonstrated. Moreover, the notion is extended to the model $Ey = X\beta$, var $y = \sigma^2 V$. Its connection with sufficiency under normality is investigated. An example illustrates the results.

21. Rao, C. R. and Boudreau, R. Diversity and Cluster Analysis of Blood Group Data on Some Human Populations. Technical Report #82-35. Center for Multivariate Analysis, November 1982

The paper discusses some systematic methods for studying differences between individuals within a population through measures of diversity and differences between populations through measures of dissimilarity.

22. Rao, C. R. Likelihood Ratio Tests for Relationship Between Two Covariance Matrices. Technical Report #82-36. Center for Multivariate Analysis, November 1982.

Likelihood ratio tests for hypotheses on relationships between two population covariance matrices Σ_1 and Σ_2 are derived on the basis of the sample covariance matrices having Wishart distributions. The specific hypotheses considered are (i) $\Sigma_2 = \sigma^2\Sigma_1$, (ii) $\Sigma_2 = \Gamma + \sigma^2\Sigma_1$, (iii) $\Sigma_2 = \Gamma + \Sigma_1$ where Γ may be n.n.d. or arbitrary and the rank of Γ is less than that of Σ_1. Some applications of these tests are given.

23. Muller, J. Existence of Unbiased Covariance Components Estimators. Technical Report #82-37. Center for Multivariate Analysis, November 1982.

The condition of Pincus (1974) for the estimability of covariance components in normal models is extended to the case of singular covariance matrices.

24. Fang, C. and Krishnaiah, P. R. On Asymptotic Distributions of Test Statistics for Covariance Matrices and Correlation Matrices. Technical Report #82-38. Center for Multivariate Analysis, December 1982.

In this paper, the authors discuss the asymptotic joint distributions of certain functions of the elements of the sample covariance matrix when the underlying distribution is a mixture of multivariate normal distributions. Application of the above distributions in studying the robustness of certain tests on correlation matrices are also discussed. The authors have also discussed the asymptotic joint distributions of certain functions of the eigenvalues of a multivariate quadratic form. Finally, applications of these results in studying robustness of certain tests on the eigenvalues of the covariance matrix are discussed when the assumption of normality is violated.

25. Yin, Y. Q. and Krishnaiah, P. R. Limit Theorems for the Eigenvalues of Product of Two Random Matrices. Technical Report #82-39. Center for Multivariate Analysis, December 1982.

In this paper, the authors showed that the spectral distribution of a sequence of the products of random matrices will tend to a distribution function in the limit as the number of variables tend to infinity.

26. Yin, Y. Q., Bai, Z. D. and Krishnaiah, P. R. The Limiting Behavior of the Eigenvalues of a Multivariate F Matrix. Technical Report #82-40. Center for Multivariate Analysis, December 1982.

Let S_1 and S_2 be random matrices of order $p \times p$ which are distributed

independently as central Wishart matrices with m and n degrees of freedom respectively. In this paper, the authors showed that the spectral distribution of $nS_1S_2^{-1}/m$ tends to limit distribution in probability.

27. Rao, C. R., Muller, J. and Sinha, B. K. Inference on Parameters in a Linear Model: A Review of Recent Results. Technical Report #83-01. Center for Multivariate Analysis, January 1983.

This paper, in th a parts, is a review of recent results on inference on parameters in a linear model. In the first part, the Gauss-Markoff theory is extended to the case when the dispersion matrix of the observable random vector is singular. In the second, robustness of inference procedures for departures in the design matrix, the dispersion matrix and distributional assumptions about the error components is considered. Finally, the third part introduces concepts of linear sufficiency and completeness in linear models, without making any distributional assumptions.

28. Krishnaiah, P. R., Lee, J. C. and Chang, T. C. Likelihood Ratio Tests on Covariance Matrices and Mean Vectors of Complex Multivariate Normal Populations and Their Applications in Time Series. Technical Report #83-03. Center for Multivariate Analysis, February 1983.

In this paper, the authors reviewed the literature on computational aspects of the distributions of the likelihood ratio statistics for testing various hypotheses on the covariance matrices and mean vectors of complex multivariate normal populations. applications of some of these test procedures in the area of inference on multiple time series in the frequency domain are also discussed. In the Appendix, the authors give tables which are useful in implementation of various likelihood ratio test statistics discussed in this paper.

29. Rao, C. R. Prediction of Future Observations in Polynomial Growth Curve Models. Technical Report #83-05. Center for Multivariate Analysis, February 1983.

The problem considered is that of simultaneous prediction of future measurements on a given number of individuals using their past measurements. Assuming a polynomial growth curve model, a number of methods are proposed and their relative efficiencies in terms of the compound mean square prediction error (CMSPE) are compared. There is a similarity between the problem of simultaneous estimation of parameters as considered by Stein and that of simultaneous prediction of future observations. It is found that the empirical Bayes predictor (EBP) based on the empirical Bayes estimator (EBE) of the unknown vector parameters in several linear models proposed by the author (Rao, 1975) has the best possible efficiency compared to the others studied. The problem of determining the appropriate degree of the polynomial growth curve is also studied from the point of view of minimizing the CMSPE.

30. Eaton, M. L. and Kariya, T. A Condition for Null Robustness. Technical Report #83-06. Center for Multivariate Analysis, March 1983.

This paper gives sufficient conditions that certain statistics have a common distribution under a wide class of underlying distributions. Invariance methods are the primary technical tool in establishing the theoretical results. These results are applied to MANOVA problems, problems involving canonical correlations, and certain statistics associated with the complex normal distribution.

31. Das Gupta, S. and Sarkar, S. K. On TP_2 and Log-Concavity. Technical Report #83-0ˉ Center for Multivariate Analysis, March 1983.

Interrelations between the TP_2 property and log-concavity of density functions have been investigated. The general results are then applied to noncentral chi-square density functions and beta density functions.

32. Sinha, B. K. Rejection of Multivariate Outliers. Technical Report #83-08. Center for Multivariate Analysis, May 1983.

An extension of Ferguson's [Fourth Berkeley Symposium on Probability and Mathematical Statistics, 1961, Volume 1] univariate normal results for rejection of outliers is made to the multivariate case with mean slippage. The formulation is more general than that in Schwager and Margolin [Ann. Statist., 1982, Vol. 10, No. 3, 943-954] and the approach is also different. The main result can be viewed as a robustness property of Mardia's locally optimum multivariate normal kurtosis test to detect outliers against nonnormal multivariate distributions.

33. Krishnaiah, P. R. Multivariate Gamma Type Distributions and Their Applications in Reliability. Technical Report #83-09. Center for Multivariate Analysis, June 1983.

In this paper, various multivariate distributions are considered to describe the failure times of the components in multicomponent systems. Some of these distributions are multivariate gamma type distributions, multivariate gamma-Weibull distributions, etc. Some methods of generating these distributions in reliability context are discussed.

34. Nguyen, T. T and Sampson, A. R. Testing for Positive Quadrant Dependence in Ordinal Contingency Tables. Technical Report #83-10. Center for Multivariate Analysis, May 1982.

Two new randomization tests are introduced for ordinal contingency tables for testing independence against strictly positive quadrant dependence, i.e., $P(X > x, Y > y) \geq P(X > x)P(Y > y)$ for all x,y with strict inequality for some x and y. For a number of cases, simulation is used to compare the estimated power of these tests versus standard tests based on Kendall's τ, Spearman's

ρ, Pearson's ξ^2, and the usual likelihood ratio test. In these cases, subsets of the alternative region are identified where each of the testing statistics is superior. The new tests are found to be more powerful than the standard tests over a broad range of the alternative regions for these cases.

35. Rao, C. R. Convexity Properties of Entropy Functions and Analysis of Diversity. Technical Report #83-11. Center for Multivariate Analysis, June 1983.

Some natural conditions which a diversity measure (variability) of a probability distribution should satisfy imply that it must have certain convexity properties, considered as a functional on the space of probability distributions. It is shown that some of the well known entropy functions, which are used as diversity measures, do not have all the desirable properties and are, therefore, of limited use. A new measure called the quadratic entropy has been introduced, which seems to be well suited for studying diversity.

Methods for apportioning diversity (APDIV) at various levels of a hierarchically classified set of populations are described. The concept of analysis of diversity (ANODIV), as a generalization of ANOVA, applicable to observations of any type, is developed and its use in the analysis of cross classified data is demonstrated. The choice of a suitable measure of diversity for the above purposes is discussed.

36. Rao, C. R. Use of Diversity and Distance Measures in the Analysis of Qualitative Data. Technical Report #83-12. Center for Multivariate Analysis, June 1983.

The paper discusses some theoretical and practical considerations in the choice of diversity and distance measures for comparing populations in terms of gene frequencies associated with various characteristics. It develops systematic methods for grouping populations by similarity in genetic diversity, apportioning diversity as between and within populations, grouping of populations by similarity in gene frequencies (cluster analysis) and testing consistency of results by using different diversity and distance measures and subsets of data. The methods can be used in the analysis of any qualitative data.

37. Rao, C. R. Inference from Linear Models with Fixed Effects: Recent Results and Some Problems. Technical Report #83-13. Center for Multivariate Analysis, June 1983.

A unified theory of BLUE from linear models is provided, where both the design matrix and the dispersion matrix of the error term may be deficient in rank. Problems of simultaneous estimation and prediction from a number of linear models are considered under a compound quadratic loss function. A generalized ridge regression estimator is proposed. Robustness of tests of significance for improper specification of the design and dispersion matrices and departure from normality is examined. The concepts of linear sufficiency

and linear minimal sufficiency are introduced and the class of BLUE's is shown to be linear minimal sufficient. Methods for model selection for predicting future observations are suggested. Finally, some properties of empirical Bayes estimators are examined.

38. Yin, Y. Q. and Krishnaiah, P. R. Limit Theorems for the Eigenvalues of the Sample Covariance Matrix when the Underlying Distribution is Isotropic. Technical Report #83-14. Center for Multivariate Analysis, July 1983.

    In this paper, the authors show that the spectral distribution of the sample covariance matrix has a limit when the underlying distribution is isotropic, and the dimension p of this distribution and the sample size n both tend to infinity but $p/n \to y < 1$.

39. Rao. C. R. Generalized Inverse of Linear Transformations: Geometric Approach. Technical Report #83-15. Center for Multivariate Analysis, July 1983.

    Generalized inverse of a linear transformation $A : \underline{V} \to \underline{W}$, where $\underline{V}$ and $\underline{W}$ are arbitrary finite dimensional vector spaces, is defined using geometrical concepts of projection without considering inner products. The inverse is uniquely defined in terms of specified subspaces $\underline{L}, \underline{M}$, and a linear transformation N satisfying some conditions. Such an inverse is called the $\underline{L}$ $\underline{M}$ N-inverse. Moore-Penrose inverse corresponds to the choice $N = 0$. Some optimization problems are considered by choosing $\underline{V}$ and $\underline{W}$ as inner product spaces. Our results extend without any major modification of proofs to bounded linear operators with closed range on Hilbert spaces.

40. Stoffer, D. S. Maximum Likelihood Fitting of Starmax Models to Incomplete Space-Time Series Data. Technical Report #83-16. Center for Multivariate Analysis, July 1983.

    In this paper we combine the spatial considerations of the space-time ARMA model and the parametrization of the ARMAX model to formulate a STARMAX model, which can be used for modeling and forecasting the dynamics of multivariate populations, which are functionally dependent upon spatial characteristics as well as time. Furthermore, due to the physical constraints imposed on a multivariate data collection system in both space and time, this model tolerates very general patterns of missing or incomplete data. As a consequence of Shumway and Stoffer (1982), the EM algorithm proposed by Dem pster et. al. (1977) is used in conjunction with modified Kalman smoothed estimators to derive a simple recursive procedure for estimating the parameters of the STARMAX model by maximum likelihood.

41. Chhetry, D, Kimeldorf, G. and Sampson, A. R. Concepts of Setwise Dependence. Technical Report #83-19. Center for Multivariate Analysis, October 1983.

This paper introduces a number of new concepts of positive and negative dependence among sets of random variables. In particular, setwise association, setwise positive upper (and lower) orthant dependence, and other related setwise concepts are defined. Properties of these various setwise dependence concepts are studied and their relationships are explored. These new concepts are particularly useful in multivariate analysis when not all interrelationships among the random variables are equally important, but rather relationships among certain sets of random variables are of primary importance.

42. Yin, Y. Q. and Krishnaiah, P. R. Limit Theorems for the Eigenvalues of Product of Large Dimensional Random Matrices when the Underlying Distribution is Isotropic. Technical Report #83-20. Center for Multivariate Analysis, October 1983.

In an earlier paper, (Journal of Multivariate Analysis, Vol. 13, 1983), the authors showed that the spectral distribution of W T has a limit, under certain conditions, when p tends to infinity and W and T are independently distributed random matrices of order p x p; here W is the central Wishart matrix In this paper, the authors generalized the above result when W is the sample sums of squares and cross products matrix and the underlying distribution is isotropic.

43. Magee, L., Ullah, A. and Srivastava, V. K. Efficiency of Estimators in the Regression Model with First Order Autoregressive Errors. Technical Report #83-21. Center for Multivariate Analysis, October 1983.

In this report, the authors consider the problem of estimating the regression coefficients in a linear regression model with first order autocorrelated disturbances when the autocorrelation is unknown. Efficiencies of various estimators of the regression coefficients are investigated.

44. Iyengar, S. Convexity of Bivariate Elliptically Contoured Distributions and Applications. Technical Report #83-23. Center for Multivariate Analysis, November 1983.

In this paper, we use a recently established partial differential equation for elliptically contoured densities to obtain approximations to probabilities of rectangles and orthants. Applications to the construction of confidence sets are given.

45. Rao, C. R., Sinha, B. K. and Mathew, T. Admissible Linear Estimation in Singular Linear Models. Technical Report #83-24. Center for Multivariate Analysis, November 1983.

The admissibility results of Rao (1976), proved in the context of a nonsingular covariance matrix, are extended to the situation where the covariance matrix is singular. Admissible linear estimators in the Gauss-Markoff model are

characterized and admissibility of the Best Linear Unbiased Estimator is investigated.

46. Kariya, T., Fujikoshi, Y. and Krishnaiah, P. R. Tests for Independence of Two Multivariate Regression Equations with Different Design Matrices. Technical Report #83-25. Center for Multivariate Analysis, November 1983.

    In this report, the authors considered a model with correlated multivariate regression equations (CMRE). Under the above CMRE model, the authors derived a locally best invariant (LBI) test for the independence of two multivariate regression equations. Null and nonnull distributions of the two other test statistics are derived. The nonnull distributions are derived under local alternatives.

47. Schaafsma, W. Standard Errors or Posterior Probabilities. Technical Report #83-26. Center for Multivariate Analysis, November 1983.

    Individual-dependent probabilities like those of the patient in clinical decision making, can more or less be regarded as estimates of posterior probabilities. Focusing on a well-defined statistical context, the asymptotic distribution of the vector of underlying estimators is obtained. These results form the basis of the decision support system POSCON, which provides estimates of posterior probabilities together with corresponding standard errors and correlations. The results can also be used in deriving approximations for the additional error rate of certain classification procedures.

48. Fujikoshi, Y. and Nishii, R. Selection of Variables in a Multivariate Inverse Regression Problem. Technical Report #84-01. Center for Multivariate Analysis, January 1984.

    We consider a multivariate inverse regression problem with an aim of estimating an unknown x vector from an observed y vector, where y and x are the vectors of p response variables and q explanatory variables, respectively. Two methods for selection of the "best" subset of response variables are given. One is based on the asymptotic mean squared error of the classical estimate. The other uses an information criterion for selection of models.

49. Ullah, A., Carter, R. A. L. and Srivastava, V. K. Sampling Distribution of Shrinkage Estimators and Their F-Ratios in the Regression Model. Technical Report #84-02. Center for Multivariate Analysis, January 1984.

    In this paper, the authors considered a general family of adaptive shrinkage estimators for the coefficients in a linear regression model. Asymptotic expansions for the distributions of these estimators are derived. The authors also compare the performance of the shrinkage estimators with ordinary least square estimators.

50. Sinha, B. K. and Chuang, J.-H. New Estimates of the Common Mean of Two

Univariate Normal Populations. Technical Report #84-03. Center for Multivariate Analysis, January 1984.

The problem of estimating the common means of two univariate normal populations with different unknown variances is considered in this paper. New estimates of the common mean, based on Stein's estimates of variances and Rao's MINQUEstimates of variances, (and their modifications), are proposed. An extensive simulation study is made to compare these and the familiar Graybill-Deal estimate from the point of view of bias and mean squared error. A suitable combination of some of these estimates is recommended.

51. Iyengar, S. A Geometric Approach to Probability Inequalities. Technical Report #84-04. Center for Multivariate Analysis, February 1984.

For elliptically contoured distributions, probability inequalities can often be reinterpreted as statements about sets on the unit sphere. We exploit this idea to give new simpler proofs of some inequalities, and to remove certain regularity conditions for them. We also prove a partial differential equation for a certain class of elliptically contoured distributions, and show its uses. Finally, we study convex contoured distributions and show that Slepian's equality does not generalize.

52. Sinha, B. K. Unbiased Estimation of the Variance of Graybill-Deal Estimate of the Common Mean of Several Normal Populations. Technical Report #84-05. Center for Multivariate Analysis, February 1984.

An identity for the chi-squared distribution is used to derive an unbiased estimate of the variance of the familiar Graybill and Deal (1959) estimate of the common mean of several normal populations with possibly different unknown variances. This result appears to be new. It is observed that the unbiased estimate is a convergent series whose suitable truncation allows unbiased estimation up to any desired degree of accuracy.

53. Rao, C. R. and Boudreau, R. Prediction of Future Observations in a Factor Analytic Type Growth Model. Technical Report #84-06. Center for Multivariate Analysis, February 1984.

Factor analytic type models are used to describe a growth process and to predict future observations. In particular, the problem of predicting the p-th observation on an individual given the previous (p-1) observations on his growth is considered when past observations are available on a number n of individuals over all the p time points. The quantity to be predicted is taken as a missing value in a (n+1) p array of observations and estimated simultaneously along with the unknown parameters of a chosen model by minimizing the sum of squares of the differences between the observations and their theoretical expectations. The methodology for the selection of an appropriate model is also discussed.

54. Wang, X. C. and Rao, M. B. Stochastic Convergence of Weighted Sums of Random Elements in Banach Spaces of Type p. Technical Report #84-07. Center for Multivariate Analysis, February 1984.

Let X_n, $n \geq 1$ be a sequence of independent random elements taking values in a separable Banach space of type p. Let a_{nk}, $n \geq 1$, $k \geq 1$ be a double array of real numbers. Some results on convergence in r^{th} - mean and strong convergence of the sequence of weighted sums $\sum_{k\geq 1} a_{nk} X_k$, $n \geq 1$ are proved generalizing some well-known results in this area.

55. Kariya, T. and Sinha, B. K. Nonnull and Optimality Robustness of Some Tests. Technical Report #84-08. Center for Multivariate Analysis, February 1984.

This paper first makes clear the invariant structure of a model for which nonnull robustness holds. Applications of this result yield the nonnull robustness of some tests for covariance structure including a test for sphericity Second, we show the optimality robustness of the LBI tests in the GMANOVA(MANOVA) problem, the problem of testing independence, and the problem of testing $\Sigma = I$. In the GMANOVA problem, a robustness property of an essentially complete class of invariant tests is also shown. Third, the UMPI test for testing the eouality of variances is shown to be optimality-robust, though it is not null robust.

56. Lau, K.-S. and Rao, C. R. On the Integral Equation $f(x) = \int_R f(x+y)d\mu(y)$, $x \geq 0$. Technical Report #84-09 Center for Multivariate Analysis, February 1984.

A general solution to the integral equation $f(x) = {}_R\, f(x+y)d\mu(y)$, $x \geq 0$ where f is a nonnegative function and μ is a σ-finite positive Borel measure on R is obtained. The equation arises in solving some characterization problems associated with the Rao-Rubin condition on damage models.

57. Das Pedadda, S. Some Results on Non-negative Estimation of Variance Components. Technical Report #84-12. Center for Multivariate Analysis, March 1984.

In this paper, the author considers the problem of obtaining non-negative estimates of the components of variance.

58. Rao, C. R. Tests for Dimensionality and Interactions of Mean Vectors Under General and Reducible Covariance Structures. Technical Report #84-13. Center for Multivariate Analysis, March 1984.

Likelihood ratio tests are derived for testing the structure of mean values in a two way classification. The most general hypothesis considered is when the mean values are subject to row and column effects and interaction has a given complexity. The observations corresponding to a row or a column classification are assumed to have an unknown dispersion (variance covariance) matrix. Two types of dispersion matrices are considered, one with

a general and another with a reducible structure. Some special cases are considered. The results of the paper provide generalizations of tests on dimensionality and interactions in a two way array of mean values considered by Fisher, Anderson, Fujikoshi, Mandel and Rao.

59. Kimeldorf, G. and Sampson, A. Setwise Dependence. Technical Report #84-14. Center for Multivariate Analysis, March 1984.

This brief expository note summarizes various approaches to measuring the degree of relationship between two or more sets of random variables, and a number of structural properties for multiple sets of random variables.

60. Nayak, T. An Analysis of Diversity Using Rao's Quadratic Theory. Technical Report #84-15. Center for Multivariate Analysis, March 1984.

This paper describes an analysis of diversity for qualitative data using Rao's quadratic entropy. The present work is a generalization of the methodology described in Light and Margolin (1971) and Anderson and Landis (1980). Measures of association are proposed based on the partitioning of the total diversity. These measures can be given interpretations similar to Goodman and Kruskal's (1954) τ - measure. Large sample tests for independence are developed and examined through simulation.

61. Nayak, T. K. On Diversity Measures Based on Entropy Functions. Technical Report #84-16. Center for Multivariate Analysis, March 1984.

Some properties and interrelationships among the diversity measures arising from the concept of entropy are discussed. Asymptotic distributions of the estimates of these measures are presented.

62. Das Gupta, A. and Sinha, B. K. Estimation in the Multiparameter Exponential Family: Admissibility and Inadmissibility Results. Technical Report #84-17. Center for Multivariate Analysis, March 1984.

This paper, in part, deals with the problem of estimating any k linear combinations of all the component means of a random vector $X(p \times 1)$ with possibly dependent coordinates and with a distribution belonging to the multiparameter exponential family. The loss assumed is sum of squared error. It is proved that if the parameter space $(H) = R^p$, the UMVUE of any two linear combinations of the means is admissible. Sufficient conditions are also given for the admissibility of arbitrary generalized Bayes estimates for $k = 2$. Next, it is observed that if (H) is not the entire R^p, admissibility of "natural" estimates may not hold even for $k = 1$. In particular, for p independent gamma distributions, a general inadmissibility theorem is proved for estimating one linear combination of the scale parameters. Finally, a sufficient condition for the admissibility of a generalized Bayes estimate for estimating p smooth scalar functions $\gamma(\theta) = (\gamma_1(\theta),\ldots,\gamma_p(\theta))$ under a sum of squared error loss has been obtained. A result of Ghosh and Meeden (AS, 1977) follows as a

corollary. Certain other applications to parametric functions more general than the means are also indicated.

63. Liang, W. Q. and Krishnaiah, P. R. Multi-stage Nonparametric Estimation of Density Function Using Orthonormal Systems. Technical Report #84-18. Center for Multivariate Analysis, March, 1984.

In this paper, the authors propose a new nonparametric method of estimation of density using orthonormal systems iteratively. In the limiting case, the mean integrated square of the estimate at each stage is less than or equal to that of the preceding stage. The new estimate is better than that of the traditional estimate based upon orthonormal functions from the point of view of the mean integrated square error in the limit.

64. Krishaiah, P. R. and Lin, J. Complex Elliptical Distributions. Technical Report #84-19. Center for Multivariate Analysis, April, 1984.

In this paper, the authors introduce a class of distributions known as complex elliptical distributions. The complex multivariate normal an complex multivarite t distributions are members of this class. Various properties of the complex elliptical distributions are studied. Finally, the robustness of certain test procedures are discussed when the assumption of complex multivariate normality is violated but the underlying distribution still belongs to the class of elliptical distribution.

65. Burbea, J. The Bose-Einstein Entropy of a Degree α and its Jensen Difference. Technical Report #84-20. Center for Multivariate Analysis, April 1984.

The Bose-Einstein entropy of degree α, B_α, is introduced and its convexity properties are studied. We show that the Jensen difference of B_α is a convex functional on its product space if and only if $2 \leq \alpha \leq 3$. A corresponding result for the Fermi-Dirac and Havrda-Charvat entropies of degree α has been established earlier by Burbea and Rao. Other related results are established.

66. Sarkar, S. and Krishnaiah, P. R. Estimation of Parameters Under Correlated Regression Equations Models. Technical Report #84-21. Center for Multivariate Analysis, April 1984.

In this paper, the authors investigated the problems connected with the estimation of parameters under correlated regression equations (CRE) model. This model is known in the econometric literature as seemingly unrelated regression equations model. The efficiencies of certain estimates of the regression coefficients are investigated under the CRE model when the underlying distribution is elliptically contoured.

67. Krishnaiah, P. R. and Sarkar, S. Nonparametric Estimation of Multivariate Density Using Laguerre and Hermite Polynomials. Technical Report #84-22. Center for Multivariate Analysis, April 1984.

In this paper, the authors investigated the problem of estimation of multivariate density using Laguerre polynomials when the variables are non-negative. It is shown that the mean integrated square error (MISE) tends to zero as the sample size tends to infinity. The rate of convergence of the MISE is also established. The authors also investigated the problem of expressing the multivariate density using mixtures of Hermite and Laguerre polynomials when only some of the variables are non-negative. The applications of the above results in the areas of pattern recognition and reliability are also discussed.

68. Liang, W. Q. and Krishnaiah, P. R. An Optimum Rearrangement of Terms in Estimation of Density Using Orthonormal Systems. Technical Report #84-23. Center for Multivariate Analysis, April 1984.

In this paper, the authors proposed an asymptotic optimum method of estimating the density function and distribution function using orthonormal functions. The method involves choosing N out of M terms for inclusion in the series to estimate the density. The criterion for the selection of the terms is based upon the magnitude of the absolute values of their coefficients.

69. Krishnaiah, P. R. and Sen, P. K. Tables for Order Statistics. Technical Report #84-25. Center for Multivariate Analysis, May, 1984.

In this paper, the authors give tables for moments of order statistics from gamma and normal populations. The authors also gave tables for the probability integrals of the maximum of correlated normal variables as well as the maximum of correlated chi-square variables with one degree of freedom.

70. Burbea, J. The Convexity with Respect to Gaussian Distributions of Divergences of Order α. Technical Report #84-26. Center for Multivariate Analysis, May 1984.

Several convexity properties for the divergences of order α, $0 \leq \alpha \leq 1$, in the family of Gaussian (normal) distributions are established. These convexity results are also formulated in terms of extended Gaussian signal and masker processes.

71. Rao, C. R. and Yanai, H. Generalized Inverses of Partitioned Matrices Useful in Statistical Applications. Technical Report #84-27. Center for Multivariate Analysis, May 1984.

Generalized inverses of a partitioned matrix are characterized under some rank conditions on the block matrices in the partitions.

72. Rao, C. R. and Nayak, T. Cross Entropy, Dissimilarity Measures, and Characterizations of Quadratic Entropy. Technical Report #84-28. Center for Multivariate Analysis, June 1984.

A unified approach is given for constructing cross entropy and dissimilarity measures between probability distributions, based on a given entropy function or a diversity measure. Special properties of quadratic entropy introduced by Rao (1982a) are described. In particular it is shown that the square root of the Jensen difference (dissimilarity measure) arising out of a quadratic entropy provides a metric on a probability space. Several characterizations of quadratic entropy are obtained.

73. Stoffer, D. S. Extraction of a Stationary Signal from Nonstationary Data Using the Bootstrap. Technical Report #84-29. Center for Multivariate Analysis, June 1984.

The bootstrap is used to approximate the standard errors of estimates of parameters and forecasts of multiple time series which are stationary signals embedded in nonstationary data. The modelling of these processes is accomplished via the state-space model along with the consideration of the various ways in which such processes can be effected by nonstationarity. Various methods of parameter estimation are discussed and a method based on the Kalman filter and the EM algorithm is suggested. It is realized that the bootstrap and the EM techniques are computationally compatible since the procedures are primarily Kalman filter and smoother runs.

74. Wang, X. C. and Rao, M. B. Some Results on the Convergence of Weighted Sums of Random Elements in Separable Banach Spaces. Technical Report #84-30. Center for Multivariate Analysis, June 1984.

Let X_n, $n \geq 1$ be a sequence of random elements taking values in a separable Banach space, A_n, $n \geq 1$ a sequence of real random variables and a_{nk}, $n \geq 1$, $k \geq 1$ a double array of real numbers. Under some conditions, we show that $\sum_{k \geq 1} a_{nk} A_k X_k$, $n \geq 1$ converges to 0 in the mean if and only if $\sum_{k \geq 1} a_{nk} f(A_k X_k)$, $n \geq 1$ converges to 0 in probability for every continuous linear functional f from the Banach space to the real line, in Section 3. The main result in Section 3 unifies many results in the literature on convergence of weighted sums of sequences of random elements. In Section 4, results on strong convergence are established. Marcinkiewicz-Zygmund-Kolmogorov's and Brunk-Chung's Strong Laws of large numbers are extended to separable Banach spaces. Using a certain stability theorem, a general result on strong convergence for weighted sums is proved from which many results in the literature follow as special cases under much less restrictive conditions.

75. Wang, X. C. and Rao, M. B. A Note on Convergence of Weighted Sums of Random Variables. Technical Report #84-31. Center for Multivariate Analysis, June 1984.

Under uniform integrability condition, some Weak Laws of Large numbers are established for weighted sume of random variables generalizing results of Rohatgi, Pruitt, and Khintchine. Some Strong Laws of Large Numbers are proved for weighted sums of pairwise independent random variables generalizing results of Jamison, Orey, Pruitt, and Etemadi.

76. Stoffer, D. S. Central Limit Theorems for Finite Walsh-Fourier Transforms of Weakly Stationary Time Series. Technical Report #84-32. Center for Multivariate Analysis, June 1984.

In this paper we utilize martingale theorems to obtain central limit theorems for the finite Walsh-Fourier transforms of various second-order stationary processes.

77. Stoffer, D. S. Estimation and Identification of Space-Time ARMAX Models in the Presence of Missing Data. Technical Report #84-33. Center for Multivariate Analysis, June 1984.

We present a method for modelling and fitting multivariate spatial time series data based on the spatial considerations of the space-time ARMA model and the parametrization of the ARMAX model. Furthermore, due to the physical constraints imposed on multivariate data collection in both space and time, the estimation and identification procedures tolerate general patterns of missing or incomplete data.

78. Liang, W.-Q. and Krishnaiah, P. R. Nonparametric Iterative Estimation of Multivariate Binary Density. Technical Report #84-35. Center for Multivariate Analysis, June 1984.

In this paper, the authors propose an iterative estimate of the multivariate density when the variables are binary in nature. Some properties of this estimate are also discussed. Finally, applications of this estimate are discussed in the areas of pattern recognition and reliability.

79. Wang, X. C. and Rao, M. B. Some Weak and Strong Laws of Large Numbers for D[0,1] - Valued Random Variables. Technical Report #84-36. Center for Multivariate Analysis, July 1984.

Pointwise Weak Law of Large Numbers and Weak Law of Large Numbers in the norm topology of D[0,1] are shown to be equivalent under uniform convex tightness and uniform integrability conditions for weighted sums of a sequence of random elements in D[0,1]. Uniform convex tightness and uniform integrability conditions are jointly characterized. Marcinkiewicz-Zygmund-Kolmogorov's and Brunk-Chung's Strong Laws of Large Numbers are derived in the setting of D[0,1]-space under uniform convex tightness and uniform integrability conditions. Equivalence of pointwise convergence, convergence in the Skorokhod topology and convergence in the norm topology for sequences in D[0,1] is studied.

80. Sarkar, Shakuntala and Krishnaiah, P. R. Tests for Sphericity Under Correlated Multivariate Regression Equations Model. Technical Report #84-37. Center for Multivariate Analysis, July 1984.

In this report, the authors considered some tests for sphericity of the error

covariance matrix under a correlated multivariate regression equations (CMRE) model. Asymptotic distributions of the test statistics associated with the above procedures are also derived.

81. Rao, C. R. Weighted Distributions Arising Out of Methods of Ascertainment. Technical Report #84-38. Center for Multivariate Analysis, July 1984.

The concept of weighted distributions can be traced to the study of the effects of methods of ascertainment upon the estimation of frequencies by Fisher in 1934, and it was formulated in general terms by the author in a paper presented at the First International Symposium on Classical and Contagious Distributions held in Montreal in 1963. Since then, a number of papers have appeared on the subject. This paper reviews some previous work, points out, through appropriate examples, some situations where weighted distributions arise and discusses the associated methods of statistical analysis.

The importance of specification of the class of underlying probability distributions (or stochastic model) in data analysis based on a detailed knowledge of how data are obtained is emphasized. Failure to take into account the conditions of ascertainment of data can lead to wrong conclusions.

82. Rao, C. R. and Shanbhag, D. N. Recent Results on Characterization of Probability Distributions: A Unified Approach Through Extensions of Deny's Theorem. Technical Report #84-39. Center for Multivariate Analysis, September 1984.

The problem of identifying solutions of general convolution equations relative to a group has been studied in two classical papers by Choquet and Deny (1960) and Deny (1961). Recently, Lau and Rao (1982) have considered the analogous problem relative to a certain semigroup of the real line, which extends the results of Marsaglia and Tubilla (1975) and a lemma of Shanbhag (1977). The extended versions of Deny's theorem contained in the papers by Lau and Rao, and Shanbhag (which we refer to as LRS theorems) yield as special cases improved versions of several characterizations of exponential, Weibull, stable, Pareto, geometric, Poisson and negative binomial distributions obtained by various authors during the last few years. In this paper we review some of the recent contributions to characterization of probability distributions (whose authors do not seem to be aware of LRS theorems or special cases existing earlier) and show how improved versions of these results follow as immediate corollaries to LRS theorems. We also give a short proof of Lau-Rao theorem based on Deny's theorem and thus establish a direct link between the results of Deny (1961) and those of Lau and Rao (1982). A variant of Lau-Rao theorem is proved and applied to some characterization problems.

83. Akamanam, S. I., Rao, M. B. and Subramanyam, K. On the Ergodicity of Bilinear

Time Series Models and Some Applications. Technical Report #84-40. Center for Multivariate Analysis, September 1984.

Existence, strict stationary and ergodicity of Bilinear Time Series Models for a given input White Noise process and parameter values is studied in detail in this paper. Using ergodicity of the model, estimation of the parameters by the method of moments is suggested and some comparisons are made with the method of least squares.

84. Bai, Z. D., Yin, Y. Q. and Krishnaiah, P. R. On Limiting Empirical Distribution Function of the Eigenvalues of a Multivariate F Matrix. Technical Report #84-42. Center for Multivariate Analysis, October 1984.

In this paper, the authors derived an explicit expression for the limit of the empirical distribution function of a central multivariate F matrix when the number of variables and degress of freedom tend to infinity in certain fashion. This distribution is useful in deriving the limiting distributions of certain test statistics which arise in multivariate analysis of variance, canonical correlation analysis, and tests for the equality of two covariance matrices.

85. Yin, Y. Q., Bai, Z. D. and Krishnaiah, P. R. On Limit of the Largest Eigenvalue of the Large Dimensional Sample Covariance Matrix. Technical Report #84-44. Center for Multivariate Analysis, October 1984.

In this paper, the authors showed that the largest eigenvalue of the sample covariance matrix tends to a limit under certain conditions when both the number of variables and the sample size tend to infinity. The above result is proved under the mild restriction that the fourth moment of the elements of the sample sums of squares and cross products (SP) matrix exist.

86. Rao, C. R. The Inefficiency of Least Squares: Extensions of Kantorovich Inequality. Technical Report #84-47. Center for Multivariate Analysis, November 1984.

Four different measures of inefficiency of the simple least squares estimator in the general Gauss-Markoff model are considered. Previous work on the bounds to some of these measures is briefly reviewed and new bounds are obtained for a particular measure.

Part II: List of Papers Published or Accepted for publication

1. Fang, C. Krishnaiah, P. R. and Nagarsenker, B. N. (1982). Asymptotic Distributions of the Likelihood Ratio Test Statistics for Covariance Structures of the Complex Multivariate Normal Distribution. Journal of Multivarite Analysis, 12, 597-611.

2. Krishnaiah, P. R. (1982). Selection of Variables under Univariate Regression Models. Handbook of Statistics (P. R. Krishnaiah, editor), Volume 2, 805-820. North-Holland Publishing Company.

3. Krishnaiah, P. R. (1982). Selection of Variables in Discriminant Analysis. Handbook of Statistics (P. R. Krishnaiah, editor), Volume 2, 883-892. North-Holland Publishing Company.

*4. Krishnaiah, P. R. and Reising, J. (1984). Multivariate Multiple Comparisons. Encyclopedia of Statistical Sciences, in press, John Wiley & Sons.

5. Fang, C. and Krishnaiah, P. R. (1984). Asymptotic Distributions of Functions of the Eigenvalues of the Doubly Noncentral F Matrix. Proceedings of the Golden Jubilee of the Indian Statistical Institute.

6. Fang, C. and Krishnaiah, P. R. (1983). On Asymptotic Distributions of Test Statistics for Covariance Matrices and Correlation Matrices. In Studies in Econometrics, Time Series, and Multivariate Statistics, (S. Karlin, T. Amemiya, and L. Goodman, editors). Academic Press.

7. Kariya, T., Fujikoshi, Y. and Krishnaiah, P. R. (1984). Tests for Independence of Two Multivariate Regression Models with Different Design Matrices. Journal of Multivariate Analysis.

*8. Kariya, T., Krishnaiah, P. R. and Rao, C. R. (1983). Inference on Parameters of Multivariate Normal Population When Some Data is Missing. Developments in Statistics, Volume 4, Academic Press.

9. Krishnaiah, P. R., Lee, J. C. and Chang, T. C. (1983). Likelihood Ratio Tests on Covariance Matrices and Mean Vectors of Complex Multivariate Normal Populations and Their Applications in Time Series. In Handbook of Statistics, Volume 3, 439-476. North-Holland Publishing Company.

*10. Sarkar, S. K., Sinha, B K. and Krishnaiah, P. R. (1983). Some Tests with Unbalanced Data from a Bivariate Normal Population. Annals of the Institute of Statistical Mathematics. Volume 35, 63-75.

11. Yin, Y. Q., Bai, Z. D. and Krishnaiah, P. R. (1983). The Limiting Behavior of the Eigenvalues of a Multivariate F Matrix. Journal of Multivariate Analysis, 13, 508-516.

*The papers with asterisks are not issued as CMA Technical Reports. All other papers listed in Part II are issued as CMA Technical Reports and the abstracts of these reports are given in Part I.

12. Yin, Y. Q. and Krishnaiah, P. R. (1983). Limit Theorems for the Eigenvalues of Product of Two Random Matrices. Journal of Multivariate Analysis, 13, 489-507.

13. Yin, Y. Q. and Krishnaiah, P. R. (1983). Limit Theorems for the Eigenvalues of the Sample Covariance Matrix when the Underlying Distribution is Isotropic. To appear in Teoriya Veroyatnostei iee Primeneniya.

14. Yin, Y. Q. and Krishnaiah, P. R. (1983). Limit Theorems for the Eigenvalues of Product of Two Random Matrices. To appear in Teoriya Veroyatnostei iee Primeneniya.

* 15. Krishnaiah, P. R. (1984). Multivariate Gamma Distribution. To appear in Encyclopedia of Statistical Sciences. John Wiley & Sons.

16. Krishnaiah, P. R. and Sen, P. K. (1984). Tables for Order Statistics. Handbook of Statistics, Volume 4, North-Holland Publishing Company.

17. Krishnaiah, P. R. and Sarkar, Shakuntala. (1984). Nonparametric Estimation of Multivariate Density Using Laguerre and Hermite Polynomials. To appear in Multivarite Analysis VI, North-Holland Publishing Company.

18. Liang, W.-Q. and Krishnaiah, P. R. (1984). Multi-stage Nonparametric Estimation of Density Function Using Orthonormal Systems. Revised version to appear in Journal of Multivariate Analysis.

* 19. Sen, P. K. and Krishnaiah, P. R. (1984). Selected Tables for Nonparametric Statistics. Handbook of Statistics, Volume 4, North-Holland Publishing Company.

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Part III: List of Other Papers

(The papers in this part do not overlap with the papers listed in Parts I and II)

1. Krishnaiah, P. R., Lin, J. and Wang, L. Likelihood Ratio Tests for the Rank of Regression Matrix When the Underlying Distribution is Complex Elliptical. To be submitted for publication.

   In this paper, the authors considered a multivariate regression matrix when the errors are distributed as complex elliptically symmetric distribution. Under the above model, likelihood ratio tests are derived to test for the rank of regression matrix. Applications of the above procedures in time series in the frequency domain are also discussed.

2. Krishnaiah, P. R. and Lin, J. Maximum Likelihood Estimators and Likelihood Ratio Test Statistics for Complex Elliptical Distributions. To be submitted for publication.

   In this paper, the authors derived the maximum likelihood estimates of the complex elliptical distributions. Also, likelihood ratio tests are derived for testing various hypotheses on mean vectors and covariance matrices. Applications of the above results in time series in frequency domain are also discussed.

3. Krishnaiah, P. R. and Sarkar, S. Principal Component Analysis Under Correlated Multivariate Regression Equations Model. To be submitted for publication.

   In this paper, the authors derived the asymptotic distributions of various test statistics useful in principal component analysis when the model consists of two correlated multivariate regression equations with different design matrices.

4. Krishnaiah, P. R., Lin, J, and Wang, L. The Likelihood Ratio Tests for Dimensionality of the Regression Coefficient Matrix When the Underlying Distribution is Real Elliptical. To be submitted for publication.

   In this paper, the authors derived likelihood ratio tests for testing the rank of regression matrix under the multivariate regression model when the underlying distribution is real elliptical.

5. Rao, M. B., Krishnaiah, P. R. and Subramanyam, K. On the Structure of Bivariate Discrete Positive Quadrant Dependent Distribution and Some Applications to Some Applications to Contingency Tables. To be submitted for publication.

   The set of all bivariate positive quadrant dependent distributions is not a convex set. In this paper, we identify certain convex subsets of this set in a natural way. In the case of discrete bivariate distributions, these convex sets

are compact, and we give a method of enumerating extreme points of the convex sets. The method of extreme point analysis helps one to compare the performance of tests for testing independence against positive quadrant dependence in the context of contingency tables.

6. Stoffer, D. S. Bootstrapping the Kalman Filter. Submitted for publication to the JASA.

   The bootstrap is proposed as a method for estimating the precision of forecasts and estimates of parameters of the Kalman Filter model. It is shown that when the system and the filter is in steady state the bootstrap applied to the Gaussian innovations yields asymptotically consistent standard errors. That the bootstrap works well with moderate sample sizes and supplies robustness against departures from normality is substantiated by empirical evidence.

7. Rosenblatt-Roth, M. The Relative Entropy of a Random Vector with Respect to Another Random Vector.

   In this report, the author gave many new results on the concept of information in probability theory and statistics.

8. Alzaid, A. A., Rao, C. R. and Shanbhag, D. N. (1984) Solutions of Certain Functional Equations and Related Results on Probability Distributions.

   Some functional equations which occur in probability theory are solved. In particular, a general version of the integrated Cauchy functional equation is considered and proved using theorems of Deny, De Finetti and Spitzer.